VERB4D Description

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1 The Fokker-Planck equation with convective terms

Convection and diffusion processes in the Earth's magnetosphere can be described by the following bounce-averaged modified Fokker-Planck equation, that includes diffusion part Schulz and Lanzerotti [1974] and convection part

$$\begin{aligned}
\frac{\partial f}{\partial t} &= \langle v_\phi \rangle \frac{\partial f}{\partial \phi} + \langle v_R \rangle \frac{\partial f}{\partial \phi} + \frac{1}{G_1} \frac{\partial}{\partial L} G_1 \langle D_{LL} \rangle \frac{\partial f}{\partial L} \qquad (1) \\
&+ \frac{1}{G_2} \frac{\partial}{\partial V} G_2 \Big(\langle D_{VV} \rangle \frac{\partial}{\partial V} f + \langle D_{VK} \rangle \frac{\partial}{\partial K} f \Big) + \\
&+ \frac{1}{G_2} \frac{\partial}{\partial K} G_2 \Big(\langle D_{KV} \rangle \frac{\partial}{\partial V} f + \langle D_{KK} \rangle \frac{\partial}{\partial K} f \Big) + \frac{f}{\tau},
\end{aligned}$$

where f is Phase Space Density (PSD); ϕ is Magnetic Local Time (MLT); R is radial distance from the Earth; L, V, K are defined as

$$\begin{aligned}
L &\equiv 2\pi \cdot B_E/\phi, \\
K &\equiv \int_{-s_m}^{s_m} [B_m - B(s)]^{1/2} ds, \\
V &\equiv \mu/(K+0.5)^2, \qquad (2)
\end{aligned}$$

where J is the second adiabatic invariant, μ is the first adiabatic invariant, B_E is the magnetic field at Earth surface. K is a pure field-geometric integral, independent on particle parameters. L will be equal to the radial distance from the Earth center to the equatorial footprint of the geomagnetic field line if the magnetic field would be adiabatically transformed to dipole field. $\langle v_\phi \rangle$ and $\langle v_R \rangle$ in the Equation (2) are bounce-averaged drift velocities; $\langle D_{LL} \rangle$, $\langle D_{VV} \rangle$, $\langle D_{KK} \rangle$, and $\langle D_{VK} \rangle$ are bounce-averaged diffusion coefficients; $G_1 = 1/L^2$ and $G_1 = (8\mu m_0 c^2)^{1/2}/(K+0.5)^2$ are the Jacobeans of the transformation from an adiabatic invariant system (μ, J, ϕ); f/τ term represents losses, where τ is electrons lifetime in a particular region.

V and K are convenient variables for numerical calculations because K is independent on particles energy and V depends weakly on particles pitch-angle. For the detailed information about solution in (V, K) coordinates see [Subbotin and Shprits, 2012].

2 Drift velocities

Bounce-averaged drift velocities are calculated following Roederer [1970]

$$\langle {}^{v_\phi}_{v_R} \rangle = \langle v_F \rangle + \langle v_{CG} \rangle = \frac{E_0 \times B_0}{qB_0^2} + \frac{1}{q\tau_\beta B_0^2} \nabla_0 J \times B_0 \tag{3}$$

where τ_b bounce time, E_0 is equatorial electric field, and B_0 is equatorial magnetic field. We use Volland-Stern electric field model with parameterization of [Leonard, 1979] and a Tsyganenko 89 [Tsyganenko, 1989] geomagnetic field model. Tsyganenko 89 magnetic field was chosen because it is based on Kp-parameterization.

At each time step, convection transport and local diffusion are calculated at each MLT and radial distance for each μ and K, and radial diffusion is calculated at each MLT as a function of L. Local energy and pitch-angle diffusion, converted into V, K space, is calculated at each MLT and radial distance. When magnetic field changes, PSD is transported radially on the computational grid conserving V and K (and therefore μ and J) in such a way that it conserves L as well.

Radial diffusion is calculated implicitly using tridiagonal numerical solutions Press et al. [1992]. We use parallel implementation of ADI method with additional stability terms to calculate local diffusion Shin and Kim [2008].

Convection is calculated using 9^{th} order upwind numerical scheme to ensure minimal numerical diffusion, with Universal Limiter addition (ULTIMATE method) to grantee that monotonic profiles stay monotonic, and Distinguishing algorithm to preserve peeks Leonard [1991]; Leonard and Niknafs [1991].

3 Diffusion coefficients

Radial diffusion coefficients consist of electromagnetic D_{LL}^{M} and electrostatic D_{LL}^{E} components that can be computed following Brautigam and Albert [2000]. [Kim et al., 2011] showed that D_{LL}^{E} parameterization from Brautigam and Albert [2000] is overestimated and produces unexpected results, so only D_{LL}^{M} component is usually used in the code.

Energy and pitch angle diffusion coefficients due to chorus day and night side waves outside the plasmasphere, hiss, lightning induced whistlers, and anthropogenic waves inside the plasmasphere are calculated using the Full Diffusion Code (FDC) Ni et al. [2008]; Shprits and Ni [2009] for all $|n| < 5$ resonance harmonic numbers. Wave activity is parameterized with Kp Shprits et al. [2007].

4 Initial conditions

As an initial conditions empty magnetosphere is usually considered for convection simulation and some sort of steady state radial diffusion solution for radial diffusion simulations. Both conditions can be used and interchanged in the code.

5 Boundary conditions

Time-dependent GOES data at geosynchronous orbit can be used to obtain PSD at the outer radial boundary, if satellite's orbit is aligned with it. If the outer boundary is farther outside, PSD variation at the outer boundary should be obtain somehow differently, e.g. with Geotail satellite data or solar wind parameterizaton.

6 Computational parameters

While ADI method has some limitations on the time step, typical convection time steps are usually much shorter and therefore it is more efficient to use ADI method as compared to UV decomposition matrix inversion method (with Lapack library) we used in previous works.

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